Signalling, Reputation and Spinoffs

Suraj Shekhar*

University of Cape Town

February 13, 2017

Internet Appendix

This internet appendix contains some proofs which were not included in the main body of the paper. Additionally, it contains other results (and a model extension) of interest which could not be included in the paper in order to keep the paper of readable length. It is organized as follows. In section 1, I discuss proofs which were omitted from the main body of the paper. In particular, I discuss the case when worker type is common knowledge in the baseline model. Section 2 contains results which were not presented in the moral hazard section of the paper. I describe conditions under which the separating equilibrium may not be highest effort equilibrium. In section 3, I discuss conditions under which the separating equilibrium outcome (as described by proposition 1 in the main body of the paper) will be the unique equilibrium outcome of the game. Section 4 considers what happens if the labour market was competitive. In particular, I look at the case where there are two principals competing for one worker. Finally, section 5 shows that there will not exist any pooling equilibrium under conditions similar to those required for proposition 1 (from the main body of the paper).

1 Proofs from main body of paper

1.1 Types are common knowledge and Principal Agent Model

I will solve the one period game beginning in period 2 first and then proceed by backwards induction. The relevant state variables [1] which affect decisions in period 2 are: One, worker formed own firm in period 1 (f)

*Email: suraj.shekhar22@gmail.com

[1]Technically, a strategy can be based on the entire history of play. However, if the customers bid something off the equilibrium path (i.e. not the expected payoff) then it does not affect the beliefs of the players about the worker type. This can be justified using Fudenberg and Tirole (1991)'s "no signalling what you don't know" condition which is built into the definition of perfect Bayesian equilibrium.

or accepted contract (nf) or play N and two, reputation of worker at the end of period 1. Let the latter be denoted by p_g^2.

Therefore, equilibrium strategies in period 2 are functions of the state variables augmented history. In particular, the strategy for principal in period 2 depends only on the value taken by the state variables:

$$s_{p,2} : (f/nf/N, p_g^2) \rightarrow \{x \in 2^{\{(s,f);\ s,f \geq 0\}};\ |x| < 3\}$$

Strategy for worker in period 2 depends upon the state variables, action taken by the principal in period 2 and own type:

$$s_{w,2} : (nf/N, p_g^2, a_p(2), T) \rightarrow \{N, acc_1, acc_2, L\}\ ;\ T \in \{G, B\}$$
$$s_{w,2} : (f, p_g^2, a_p(2), T) \rightarrow \{N, acc_1, acc_2, S\}\ ;\ T \in \{G, B\}$$

Strategy for customer in period 2:

$$s_{c,2} : (f/nf/N, p_g^2, a_p(2), a_w(2)) \rightarrow \{b;\ b \in \mathbb{R}_+\}$$

Since I have mistakes in the model and mistake probabilities go to zero, the relevant equilibrium concept is trembling hand perfect equilibrium.

1.1.1 In Period 2

Claim 1. *When worker type is common knowledge, if the worker formed a firm in period 1, then, in period 2 any contract which is IC for the worker to accept is not IR for the principal.*

Proof. Let the worker be G type. Then the worker gets V as project price from the customers[2]. To make it incentive compatible for the worker to accept his contract, the principal will have to pay atleast V as wages. The customers pay the same price (V) for the project when the firm is owned by the principal because their bidding simply depends upon the worker type. Thus, if the worker accepts his contract, the principal's payoff will be at most $V - V - R_p < 0$. If the worker type is B, then it is not worthwhile for the principal to form the firm even if he can pay the worker zero wages. This follows from the assumption $\lambda_b V \leq R_p$. □

Corollary 1. *In any SPNE, if the worker type is G, the principal will offer the contract* $\{0, 0\}$ *in period 2 if the worker did not accept a contract in period 1.*

[2]The customers are risk neutral and are competing with one another to get the services of the firm. So they will both bid their expected value (V) of the project and one will be randomly selected as the winner.

Proof. If the worker formed his own firm in period one then the principal forms his firm in period 2 only if the worker makes a mistake. Therefore, he offers the lowest paying contract. If the worker had chosen N in period 1, then we know that the worker can only get a negative payoff by forming his own firm in period 2 ($R_w > V$). Therefore, he will accept a zero wage contract. [3] It is IR for the principal to offer the zero wage contract because $R_p < V$. Obviously, it is not optimal for the principal to offer higher wages. □

Corollary 2. *In any SPNE, if the worker type is B, the principal will offer no contract in period 2 if the worker did not accept a contract in period 1.*

Proof. The highest payoff the principal could get is when he does not have to pay any wages. This payoff is $(\lambda_b V)$. This does not compensate for the cost of setting up the firm R_p since by assumption we have $\lambda_b V < R_p$. □

Corollary 3. *If worker type is known and the worker has formed his firm at period 1, then the principal can never form a principal-worker firm in period 2.*[4]

Claim 2. *In any SPNE, the principal will offer the contract* $\{0,0\}$ *in period 2 if the worker accepted a contract in period 1.*

Proof. Since the payoff from forming a firm in period 2 is negative for worker he is willing to accept zero wages. The principal is willing to offer this contract since he has already invested R_p in period 1 and therefore faces no additional cost in offering this zero wage contract. □

Thus the payoffs in period 2 can be summarized as follows (first coordinate is principal's payoffs and the second is worker's payoff.):

Table 1: Payoffs in Period 2

Table 2: Payoff if contract accepted in Period 1

$(Principal, G)$	$(Principal, B)$
$(V, 0)$	$(\lambda_b V, 0)$

Table 3: Payoff if Spinoff in Period 1

$(Principal, G)$	$(Principal, B)$
$(0, V)$	$(0, \lambda_b V)$

Table 4: Payoff if N in Period 1

$(Principal, G)$	$(Principal, B)$
$(V - R_p, 0)$	$(0, 0)$

[3]The worker could also choose N. However, we have assumed that if the worker is getting atleast zero from any other action choice he will choose that action.

[4]Unless there is a mistake.

1.1.2 In Period 1

Claim 3. *In the unique subgame perfect equilibrium of the game, when the worker type is B, the principal offers the worker a contract with zero wages ($\{0,0\}$) in period 1 and the worker accepts. If the worker type is G then the principal offers the contract $\{V-R_w+\delta V,0\}$ in period 1 and it is accepted by the worker. In period 2, (along the equilibrium path) the principal offers a zero wage contract which is accepted by the worker.*

Proof. By assumption, it is not IR for B worker to invest in forming own firm at period 1. Therefore, the principal will offer zero wage contract which will be accepted. Note that it is IR for the principal to offer this contract by the assumption $\lambda_b V(1+\delta) > R_p$. It will not be incentive compatible to offer more. When the worker type is G the worker gets the payoff $V-R_w+\delta V$ by investing R_w and starting his own firm. The principal must offer him this amount to dissuade him from leaving. Offering more will not be optimal. Moreover, it is known that the principal cannot commit to offering more than zero wages in period 2 if the worker does not form a firm in period 1. Therefore, the principal offers him a contract which pays him $V-R_w+\delta V$ upon success [5]. Offering this contract is also IR for the principal as the principal's payoff is $V-(V-R_w+\delta V)-R_p+\delta V = R_w-R_p(>0)$. □

Corollary 4. *The worker never forms his own firm if worker type is known.*

Thus, equilibrium payoffs to the principal and worker in the game when types are known are given by the following matrix where (the first coordinate refers to the principal's payoff)

Table 5: Payoffs if Types are Known

$(Principal,G)$	$(Principal,B)$
$(R_w-R_p,V-R_w+\delta V)$	$(\lambda_b V(1+\delta)-R_p,0)$

2 Additional Results from Moral Hazard Section

Claim 4 and claim 5 together serve to show that the spinoff equilibrium may not always produce the most effort.

Claim 4. *There exists $\overline{R}$ such that if $R_w > \overline{R}$ and the following holds*

$$\frac{\frac{1+\delta\beta}{1-\beta}+R_w}{1+\delta\beta} < V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)} < R_p$$

[5] Since G type always succeeds at V, there is no point in offering him any payoff for failure

then there exists p' such that if $p_g < p'$, there exists a separating equilibrium where the G worker's strategy (along the equilibrium path) is to leave and form his own firm in period 1 and the B worker's strategy (along the equilibrium path) is to accept a contract in period 1. B type worker exerts $e = 1$ in both periods and G worker exerts $e = 1$ in period 1 and $e = 0$ in period 2. The principal offers the contract $\{(1/(1-\beta)), 0\}$ to the worker in period 1 and the same in period two if the worker accepts in 1.

Before I describe the details of the proof, I present the basic ideas here. First, why doesn't the B type worker copy G type's strategy? The B type worker does not try to imitate the G type worker and play L because of the following reason: the G type worker is supposed to exert $e = 1$ after leaving. This means that the G type worker will succeed in period 1. Thus, if a worker fails (and the B type worker can fail even if he puts in maximum effort), he will be recognized as a B type worker and get paid less tomorrow. This, along with an upper bound on V, reduces the expected future wages of the B type worker enough for him to not copy the G type worker's strategy. Second, why does the G type worker put in maximum effort in period 1? The G type worker does not deviate and put in less effort for fear of failing and being thought of as a B type worker (which would reduce his payoff in period 2). Third, why can't the principal stop the G type worker from leaving? If the principal offers a high wage contract to attract the good worker, both type workers will accept it leading to a pooling equilibrium. If the belief about the worker's type is very low then the principal cannot expect huge prices from the customers (even if he offers lucrative contracts which makes the worker put in maximum effort). This means that the principal is unwilling to offer the high wage contract needed to stop the G type worker from leaving. Finally, why does the B type worker put in $e = 1$ in both periods? Given the conditions (specially lower bound on V) the principal finds it worthwhile to offer wages high enough to induce $e = 1$ from a B type worker.

Proof of claim 4

Consider the following strategies in period 2:

For Principal :

$$s_{p,2}(nf,p_g^2)=\{\frac{1}{1-\beta},0\}\ ;\ p_g^2<\frac{V-\frac{1}{(1-\beta)^2}+\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(nf,p_g^2)=\{0,0\}\ ;\ p_g^2\geq\frac{V-\frac{1}{(1-\beta)^2}+\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N,p_g^2)=\ \phi$$

$$s_{p,2}(f,p_g^2)=\ \phi$$

For Worker :

$If\ worker\ had\ formed\ firm\ in\ period\ 1\ only\ one\ case\ is\ relavent\ since\ V<R_p$

$$s_{w,2}(f,p_g^2,\phi,G/B)=\ S,\ e=0$$

$If\ worker\ had\ not\ formed\ firm\ in\ period\ 1:$

$$s_{w,2}(nf/N,p_g^2,\{\{s_1^2,f_1^2\},\{s_2^2,f_2^2\}\},G)=\ Choose\ (contract,effort)\ maximizing\ payoff$$

$$s_{w,2}(nf/N,p_g^2,\{\{s_1^2,f_1^2\},\{s_2^2,f_2^2\}\},B)=\ Choose\ (contract,effort)\ maximizing\ payoff$$

$$s_{w,2}(nf/N,p_g^2,\phi,G/B)=N$$

Consider the following strategies in period 1:

For Principal :

$$s_p(\phi)=\{\{\frac{1}{1-\beta},0\}\}$$

For Worker :

$WLOG\ let\ contract\ 1\ be\ better\ than\ contract\ 2\ for\ G\ worker$

$$s_w(\{\{s_1^1,f_1^1\},\{s_2^1,f_2^1\}\},G)=acc_1,e=1\ ;\ -1+s_1^1+\delta\frac{1}{1-\beta}=max\{-1+s_1^1+\delta\frac{1}{1-\beta},\beta s_1^1+\delta\frac{1}{1-\beta}\}\ and$$

$$-1+s_1^1+\delta\frac{1}{1-\beta}\geq-1+V-R_w+\delta\beta V$$

$$s_w(\{\{s_1^1,f_1^1\},\{s_2^1,f_2^1\}\},G)=acc_1,e=0\ ;\ -1+s_1^1+\delta\frac{1}{1-\beta}\neq max\{-1+s_1^1+\delta\frac{1}{1-\beta},\beta s_1^1+\delta\frac{1}{1-\beta}\}\ and$$

$$\beta s_1^1+\delta\frac{1}{1-\beta}\geq-1+V-R_w+\delta\beta V$$

$$s_w(\{\{s_1^1,f_1^1\},\{s_2^1,f_2^1\}\},G)=L,e=1\ ;\ else.$$

$$s_w(\{\{s_1^1,f_1^1\},\{s_2^1,f_2^1\}\},B)=\ Choose\ (contract,effort)\ maximizing\ payoff$$

$$s_w(\phi,B)=N$$

$$s_w(\phi,G)=L,e=1$$

Let us prove by backward induction that the above strategies constitute a perfect bayesian equilibrium under the conditions imposed in the theorem.

In period 2, consider the incentives of the worker. If the history is such that the worker has formed a firm of his own in period 1, then the worker is not offered any contracts in period 2 since the principle can never find this to be individually rational as $V < R_p$. In this case, it is clear that the best action for the worker is to play S and then exert zero effort. Zero effort is exerted since all payments are received by the worker before his effort choice and the game ends in period 2. If the history is one in which the worker had played N in period 1, then it is not individually rational for either the worker or the principal to form a firm in period 1, therefore the only on the equilibrium path response is to play N again. In a history in which the worker had accepted a contract in period 1, it is not individually rational for the worker to think about playing L, therefore the best action choice is to choose the most profitable (contract, effort) tuple.

Now let us consider the incentives of the principal in period 2. Suppose the history is one in which the worker did not accept the principal's contract in period 1. It is not IR for the principal to form the firm in period 2. Therefore the principal's optimal action is to offer no contract in period 2. If the worker had accepted a contract with the principal in period 1, then the principal has two choices - he can either offer a contract $\{(1/(1-\beta)), 0\}$ which is the minimum amount required for a worker of either type to put in effort 1[6] or he can offer a contract which pays nothing and get zero effort from the worker. Note that any contract which offers something in the middle is not optimal. We need the following conditions to make it IR and IC for the principal to offer the contract $\{(1/(1-\beta)), 0\}$ and extract maximum effort:

$$
\begin{aligned}
& Payoff\ from\ \{\frac{1}{1-\beta}, 0\} \ > \ Payoff\ from\ \{0,0\} \\
& \Leftrightarrow [p_g^2 + (1-p_g^2)(\beta\lambda_b + (1-\beta))](V - \frac{1}{1-\beta}) > [p_g^2\beta + (1-p_g^2)\beta\lambda_b]V \\
& \Leftrightarrow p_g^2 \leq \frac{V(1-\beta^2) - \beta\lambda_b - (1-\beta)}{\beta(1-\lambda_b)}
\end{aligned} \tag{1}
$$

It is clear that if R_w is high enough and $V > (((1+\delta\beta)/(1-\beta)) + R_w)/(1+\delta\beta)$, then $V(1-\beta^2) - \beta\lambda_b - (1-\beta) > 0$. In any separating equilibrium where the G worker forms his own firm in period 1, $p_g^2 \approx 0$. Thus, 1 is satisfied.

Therefore, under a separating equilibrium, the payoffs in period 2 are as follows. The principal expects to get a payoff of $(\beta\lambda_b + 1 - \beta)(V - (1/(1-\beta)))$ in period 2 if the worker accepts the contract in period 1. If the worker does not accept contract in period one, the principal expects zero payoff in period 2. A G type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1 + (1/(1-\beta))$ if he

[6]This is regardless of worker type i.e. even if the worker type was known, for any type of worker, this contract would be an optimal contract to induce maximum effort.

had accepted a contract in period 1. A B type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1+(\beta\lambda_b+(1-\beta))\frac{1}{1-\beta}$ if he had accepted a contract in period 1.

Let us now consider the incentives for the worker in period 1. Given the payoffs in period 2, G type worker cannot do better. In any separating equilibrium the G worker must put in maximum effort in period 1 after playing L. We need the following condition to ensure that this is incentive compatible: $V > \frac{1}{\delta\beta(1-\beta)(1-\lambda_b)}$. Given that the G worker will put in full effort after separation, it is IR for the G type worker to choose to play L if $V > \frac{1+R_w}{1+\delta\beta}$. It is clear that if R_w is high enough and $V > \frac{\frac{1+\delta\beta}{1-\beta}+R_w}{1+\delta\beta}$, then these conditions are satisfied. It is not IR for the B type worker to leave if the following condition holds: $V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$. This holds according to the conditions in this claim, therefore, his strategy to choose the best (contract,effort) combination is optimal. So till now we have that if R_w is high enough and $\frac{1+R_w}{1+\delta\beta} < V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$, then the following hold: 1. IR for G to play L, 2 Not IR for B to play L, 3 If G plays L, then first period effort is 1, 4 B will accept $\{\frac{1}{1-\beta},0\}$ and put in effort 1 in both periods and 5 G worker will play $e=0$ in period 2 after L in period 1.

Consider incentives of G worker in period 1 after the principal offers a contract $\{s,0\}$.

Payoff from accept and $e=1$ = $-1+s+\delta(-1+\frac{1}{1-\beta})$

Payoff from accept and $e=0$ = $\beta s+\delta(-1+\frac{1}{1-\beta})$

Payoff from L = $-1+V-R_w+\delta\beta V$

Therefore, G worker will accept and play $e=1$ if

$$s \geq \frac{1}{1-\beta} \ \& \ s \geq V-R_w+\delta(\beta V-\frac{\beta}{1-\beta})$$

G worker will accept and play $e=0$ if

$$s < \frac{1}{1-\beta} \ \& \ s \geq \frac{1}{\beta}[-1+V-R_w+\delta(\beta V-\frac{\beta}{1-\beta})]$$

G worker will play L if

$$s < \frac{1}{\beta}[-1+V-R_w+\delta(\beta V-\frac{\beta}{1-\beta})] \ \& \ s < V-R_w+\delta(\beta V-\frac{\beta}{1-\beta})$$

Now:

$$
\begin{aligned}
&V > \frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1+\delta\beta} \\
&\Leftrightarrow V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) > \frac{1}{1-\beta} \qquad (2)
\end{aligned}
$$

and we can show that

$$
V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) < \frac{1}{\beta}[-1 + V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})] \qquad (3)
$$

So by 2 and 3, if the principal offers $s = \frac{1}{1-\beta}$ or below, G worker will play L. If R_w is high enough then we have $\frac{\frac{1+\delta\beta}{1-\beta}+R_w}{1+\delta\beta} < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$. So we can pick a V such that $\frac{\frac{1+\delta\beta}{1-\beta}+R_w}{1+\delta\beta} < V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$.

The principal now has three choices which may be optimal. One, offer a zero wage contract which the B type worker will accept and put zero effort. Two, offer the contract $\{\frac{1}{1-\beta}, 0\}$ which the worker will accept if he is B type and put in full effort. Three, offer the contract $\{V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}), 0\}$ which both type workers will accept and put in full effort[7]. The principal offers the contract $\{\frac{1}{1-\beta}, 0\}$. This is less than the minimum needed to attract the G type worker. Clearly, the principal will be willing to offer no more than this if p_g is low enough i.e. if the principal believes that the customers are convinced that the worker must be B type if he accepts a contract then it would not be optimal for the principal a high paying ($V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})$ is strictly bigger than $\frac{1}{1-\beta}$) contract to attract both types of workers[8]. Thus, we can find the required p'. The principal does not offer a lower pay contract because V is high enough for him to want to induce high effort. Given this contract, the G worker's optimal choice is to form his own firm in period 1.

Claim 5. *Suppose the conditions required for the previous claim hold. Then, there exists a p'' such that if $p_g \in [0, p'']$, there exists a pooling equilibrium in which a worker of any type chooses to accept a contract in period 1 and 2 and puts in effort=1 in both periods.*

Proof. If p_g is low and a worker of any type is expected to accept the contract, then, if a worker deviates and plays L it would be viewed by the customers as a mistake. Since mistake probabilities are the same, the worker will have a reputation of p_g. If p_g is low, it would not be individually rational to play L (this follows from $V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$). Therefore, the strategy of both type workers must be to accept the best contract if the belief is that they will.

[7] 3 ensures that that offering a contract which would make the G worker accept and put in zero effort is not optimal for the principal.

[8] I omit the formal proof of this part since the idea is the same as that in propositions in the main body of the paper.

Consider the following strategies in period 2:

$$\textbf{For Principal}:$$

$$s_{p,2}(nf, p_g^2) = \{\frac{1}{1-\beta}, 0\} \; ; \; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(nf, p_g^2) = \{0, 0\} \; ; \; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

$$\textbf{For Worker}:$$

$$If\ worker\ had\ formed\ firm\ in\ period\ 1\ only\ one\ case\ is\ relavent$$

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, \; e = 0$$

$$If\ worker\ had\ not\ formed\ firm\ in\ period\ 1:$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_{w,2}(nf/N, p_g^2, \phi, G/B) = N$$

Consider the following strategies in period 1:

$$\textbf{For Principal}:$$

$$s_p(\phi) = \{\{\frac{1}{1-\beta}, 0\}\}$$

$$\textbf{For Worker}:$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G/B) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_w(\phi, G/B) = N$$

The above strategies would give us the pooling on contract equilibrium. □

Corollary 5. *Given worker type, the above pooling equilibrium is weakly better than the separating equilibrium described in claim 4 since it gives a higher probability of success of the project in each period.*

Proof. For any type of worker, the pooling equilibrium guarantees full effort in both periods. Whereas, the separating equilibrium does not give us full effort from the G type worker in period 2. □

3 Uniqueness of Equilibrium

Is it possible to get the equilibrium outcome in the separating equilibrium[9] as the unique equilibrium outcome? I show that if the following two assumptions hold along with the conditions required in proposition 1 (in the main body of the paper), then the separating equilibrium outcome is the unique outcome of the game. Assume the following:

- Assumption A - If a worker leaves to form his own firm in period one then he may not sign a contract with the principal in period 2.
- Assumption B - Given an equilibrium, if a mistake (non-equilibrium action) gives the worker strictly negative payoffs for all possible beliefs that the customers may have about the worker who makes that mistake, then the probability of such a mistake is much lower than the probability of a mistake where the worker may get positive payoffs for some beliefs. In particular, I assume the following mathematical specification[10] - if a mistake gives the worker strictly negative payoffs for all possible beliefs that the customers may have about the worker who makes that mistake, then the probability of such a mistake is ε^2 (independent of type). Else, the probability of the mistake is ε (independent of type)

Essentially, if Assumption B and the conditions required in proposition 1 (in the main body of the paper) hold, then the only equilibrium outcomes possible are separating equilibria where the G worker forms his own firm in period 1 and the B worker's strategy is to accept a contract in period 1. Adding assumption A to the fray makes sure that the play in period 2 following separation corresponds to the equilibrium in which both worker types choose to stay with the new firm instead of accepting a contract. An intuitive idea of these claims follows.

Assumption A above guarantees that the second period play following the worker forming his own firm in period one and then succeeding is that of pooling on S[11]. The separating equilibrium described before has the feature that the good type worker forms his own firm in period 1 whereas the bad type worker would accept a contract in period 1. In period 2, if the history is one where the worker has formed a firm and then succeeded at the project, the worker's choice along the equilibrium path is to stay at his own firm. Assumption A eliminates equilibria of the following form[12] : the good type worker forms his own firm in period one but

[9] Where the good type worker always forms a spinoff and then remains at the spinoff in period 2 while the bad type worker's strategy is to accept a contract and stay with the principal in period 2 as well.

[10] Any other specification where one mistake is an order more likely than the other will also work

[11] Remember, worker can make mistakes. So even in a separating equilibrium the belief about the worker who forms his own firm goes close to 1 but is not equal to 1.

[12] For a formal statement of this look at the online appendix.

signs a contract with the principal in period 2. The bad type worker accepts a contract in period one and does the same in period 2. In case the bad type worker makes a mistake in period one and forms his own firm, then in period 2, this worker stays with his own firm.

Assumption A is not an implausible one. It would be odd for the customers to hold beliefs that a worker who has invested a lot of money into building his own firm in period 1, decides to forfeit that investment and accept a contract from the principal in period 2. Such beliefs would be even more implausible if the worker was successful in period one and has a very high reputation (since in this case the principal could not be offering more than what the worker could have earned by having a strategy of staying with his own firm). Moreover, we can obtain similar results if we simply assume that $V < R_p$. In this case, it would not be individually rational for the principal to form the firm in period 2. This assumption would be intuitive in environments where the fixed cost of starting a firm is recovered only after several periods of operation (as would be the case in many industries). Other authors have also used such assumptions to simplify analysis, example (Golan (2009)).

Now lets assume the conditions required in proposition 1 (in the main body of the paper). In particular, $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ combined with assumption B helps us remove equilibria of the kind where both workers accept a contract in period 1[13]. An intuitive idea of such an equilibrium is as follows: Principal offers zero wage contract in period 1 and 2. p_g is low. Both type workers accept this contract in both periods. This is because if G type worker leaves to form his own firm, it will be considered a mistake and his reputation will be low (assuming $\alpha = 1 \Rightarrow$ equal probabilities of mistake). Forming his own firm with a low reputation is not IR for the G worker so he accepts the zero wage contract. However, with assumption B, if a worker forms his own firm in period one then the belief about that worker will become close to 1 (ratio of mistake probabilities is $\frac{1}{\varepsilon}$ and $\varepsilon \to 0$). This is because only the G worker can potentially gain from forming his own firm. The B type worker will always get negative payoffs. Now, this deviation becomes profitable to the G worker as his payoffs become approximately $V - R_w + \delta V$ (using assumption A) and therefore we can eliminate equilibria of this kind.

Assumption B is used to restrict beliefs about a worker who makes the costly investment and forms his own firm. Combined with $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$, this assumption implies that since only the G type worker can possibly gain by forming his own firm, the beliefs about the type of a worker who plays L in period 1 must be close to 1. This assumption is most natural when the difference between the expected future payoff of a G and B worker is large. If this difference is small, then this assumption becomes a strong one. The difference is inversely proportional to λ_b. Therefore, assumption B is mild if used when the difference in abilities of the

[13] We describe such an equilibrium in the the online appendix.

two types are large. The next proposition formally describes the conditions needed to obtain the separating equilibrium as the the unique equilibrium.

Proposition 1. *Let Assumption A and B hold. Let* $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. *If* $p_g < \frac{R_p-\lambda_b R_w+(\lambda_b V(1+\delta)-R_p)}{R_w-\lambda_b R_w+(\lambda_b V(1+\delta)-R_p)}$, *there exists a separating equilibrium in which the G worker leaves to form own firm in period one and the B worker accepts a zero wage contract in period 1. Moreover, this is the unique equilibrium outcome of the game.*

Proof. $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda)}$ guarantees that the only equilibrium outcomes possible in period 1 are those in which either the G worker leaves and the B worker accepts a contract or those in which both type workers accept a contract. The above discussion eliminates the latter. Assumption A guarantees the play in period 2 is same as that in the separating equilibrium described before. □

4 Competitive labour market

In this section, I will consider an environment in which the market for labour is competitive. There will be two principals offering contracts to attract the worker. It will be interesting to contrast the results obtained in the previous section with the ones in this section. In particular, it is not obvious if it is easier or harder for the G worker to separate and signal his type in a competitive labour market as compared to the monopsonistic labour market analyzed in the previous section. On one hand, competition between principals will lead to the workers getting higher wage contracts which reduces the incentives to separate. On the other hand, competition reduces the payoffs a principal may earn in equilibrium. This makes it less lucrative for any principal to prevent a worker from leaving by offering him high wage contracts. Additionally, the results in this section should shed some light on the kind of markets in which we are more likely to have signalling driven spinoffs.

The model remains the same as the baseline model except for the following change. There are two principals and one worker. In period 1, both principals offer competing contracts simultaneously. The worker decides between accepting either contract, forming his own firm or doing nothing. In period 2, both principals offer their contracts simultaneously again. Then, the worker decides between staying with his own firm (if he had formed his own firm in period 1), accepting a contract offered by one of the principals and doing nothing. If the worker had accepted a contract with a principal in period 1 then the worker chooses between the best contract, forming his own firm and the action N after observing the contracts offered by both principals. I will look for equilibria where the principals play a symmetric equilibrium.

4.1 Types are Common Knowledge

In this subsection, consider the case where there are two principals competing for one worker whose type is known by all players. I will assume that the principals offer just one contract each. Since there is no uncertainty about type, the only reason the principals may want to offer more than one contract is that they hope to catch the worker in a bad contract if he makes mistake. However, since mistake probabilities are almost zero, this is not be a big assumption and can be motivated by assuming a small cost to offering every additional contract.

The main result is discussed below. As in the complete information case with one principal, the worker never chooses to form his own firm in equilibrium. However, in this environment, competition between the principals causes their profits to go to zero in equilibrium and the workers get much more in wages.

I assume that when the worker type is G, the principals do not offer any reward for failure. Since the good type worker never fails, this is an innocuous assumption.

Proposition 2. *If the worker type is known and the worker is of type G then both principals play a symmetric mixed strategy in period 1: principals bid a contract which pays less than the contract* $\{x,0\}$ *with probability* $G(x)$ *where* $x \in [\underline{x},\overline{x}]$*. The G type worker accepts the better of the two realizations of contracts offered from the above distribution.*

Proof. Details (including a description of $G(x),\underline{x},\overline{x}$) are in the appendix (proposition 6 in the appendix.). Once again, since the principals have a lower cost of firm formation, they are always able to offer a contract which the worker cannot refuse. To show that the worker will accept the contract it is sufficient to show that the worker accepts the lowest wage contract. The lower bound of the support of the distribution $G(x)$ is such that this holds. □

Thus, if the worker type is G, then in the full information case, the worker will not form his own firm and the expected payoff to the worker will be approximately $V - R_p + \delta V$. The principals get an expected payoff of zero. Similarly it can be shown that if the worker type is B, then in the full information case, the worker will not form his own firm. The payoff to the worker will be $\lambda_b V - R_p + \delta \lambda_b V$ and competition between the principals will force their expected payoff to zero.

Thus payoffs in the equilibrium in the full information case can be summarized as follows. First coordinate is payoff for principal 1, second is payoff for principal 2, and third is payoff for worker.

This result is not surprising. As in the previous section, if worker types are known and the principals have a cost advantage in firm formation then they are always able to offer the worker a contract which the

Table 6: Expected Payoffs if Types are Known and Competitive Labour Market

$Principal\ 1, Principal\ 2, G$	$Principal\ 1, Principal\ 2, B$
$(0,0,V-R_p+\delta V)$	$(0,0,\lambda_b V(1+\delta)-R_p)$

worker accepts. Moreover, competition for the worker leads to zero payoff for the principals in equilibrium. The worker gets the entire surplus.

4.2 Types are Private Knowledge

From the previous section, we have learnt that just as in the game with no competition in the labour market, when there is full information about the worker's type, the worker never forms his own firm. We will show in this section that this result does not stand when the worker's type is not known. The intuition for this result will be the same as before. The novel result from this section will be the result which compares the two environments and finds that while a separating equilibrium is possible in both environments, we require stronger conditions to get the separating equilibrium when there is competition for the worker. The intuition for this result is that competition between the principals bids up the wages of the worker which makes the opportunity cost of firm formation much higher. The result that workers with higher wages may be less likely to become entrepreneurs than workers with lower wages has also been shown by papers like Evans and Leighton (1989).

The worker's type is his private information in this subsection. The main results are presented below. A detailed analysis can be found in the appendix. The first result provides sufficient conditions under which we can have a separating equilibrium where the good type worker forms his own firm in period one while the bad type worker accepts a contract in period one.

4.2.1 Separating Equilibrium under Competitive Labour Market

Proposition 3. *Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $R_p > \lambda_b R_w$. There exists a p_c such that if $p_g \in (0,p_c)$ then there exists a separating equilibrium where the G worker forms a spinoff in period 1 and the B worker accepts the best of the contracts offered by the two principals.*

Proof. The details of the proof are in the appendix (proposition 9 in appendix). The intuitive idea is the same as that in proposition 1 in main body of this paper. The customers will pay according to their expected payoffs. If the belief about the worker who accepts the contract is low then it may not be optimal for the principal to offer a high wage contract to stop the G worker from leaving and forming own firm. The B worker does not try to imitate the action of the G worker since he realizes that his expected future payoff is much lower than that of the G worker because of the possibility of failure which would reveal his type. Note, however that we need an additional condition of $R_p > \lambda_b R_w$ here. This is because the outside option for the principals has changed. In the baseline principal-worker model, even if the G worker's strategy was to leave in period 1, the principal could get positive payoffs by extracting all rents if the worker happened to be B type. In a separating equilibrium, when the labour market is competitive, competition forces the principals to offer contracts which would give them zero rent even if the worker turns out to be B type. Thus, to deviate, all the principal needs is a contract which gives him positive payoffs (as opposed to the baseline case where, to get a deviation, the principal would have required a deviation which pays more than the payoff from getting B type worker). Therefore, the principals are more likely to offer a higher pay contract in this environment and thus, to preclude the principals from offering any contract which would prevent G worker from leaving, we need the condition that the principal's cost of setting up the firm is not too low. □

We can get the above equilibrium to be the unique equilibrium outcome under conditions similar to those described for uniqueness in the appendix of the main body of the paper. Since there is no new intuition, I omit the statement and proof here. Next we compare the conditions needed to get separation in the two environments discussed so far. In particular it would be interesting to know which environment requires stronger conditions i.e. in which environment are spinoffs less likely.

4.2.2 Separating in Competitive vs Non-Competitive Labour Market

One issue with comparing conditions needed for the separating equilibrium in the two environments is that under general parameters there are multiple equilibria in these environments. However, to be able to compare the two environments we need to know exactly which equilibrium will be played in atleast one environment[14]. Therefore, in this subsection, I assume assumption A,B hold and the sufficient conditions needed for a separating equilibrium described in proposition 1 (in main body of this paper) also hold i.e. $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $p_g < \frac{R_p-\lambda_b R_w+(\lambda_b V(1+\delta)-R_p)}{R_w-\lambda_b R_w+(\lambda_b V(1+\delta)-R_p)}$. Therefore, by the uniqueness proposition (proposition 1), we know that the unique equilibrium outcome in the principal agent model is a separating equilibrium where the good type worker's strategy (along the equilibrium path) is to form his own firm in period 1 and stay with the new firm in period 2 and the bad type worker's strategy is to accept the best contract offered in either periods. The next proposition points out that under some conditions, there may exist a separating equilibrium under the principal worker environment but not when there are two principals competing for one worker. This provides the intuition for the result that signalling one's ability via firm formation may be harder when the market for labour is competitive.

[14]The alternative is to compare the sets of equilibria somehow.

Proposition 4. *If* $p_g \in (\frac{R_p-\lambda_b R_w}{R_w-\lambda_b R_w}, \frac{R_p-\lambda_b R_w+\lambda_b V(1+\delta)-R_p}{R_w-\lambda_b R_w+\lambda_b V(1+\delta)-R_p})$*, then there does not exist a separating equilibrium where the G worker forms his own firm and the B type worker accepts a contract.*

Proof. Detailed proof is in the appendix. A sketch of the proof is as follows. We will prove by contradiction. Suppose there exists an equilibrium where the G type worker forms his own firm in period one and the B type worker accepts the best contract in period 1. Competition and symmetry of principals will drive the expected payoff of the principals to zero.

Given any equilibrium that will be played in period 2, we can show that if a principal deviates in period 1 and offers a contract which would give the G worker a payoff of $V - R_w + \delta V$ in total (over two periods), then that principal will make positive profits if $p_g > \frac{R_p-\lambda_b R_w}{R_w-\lambda_b R_w}$. This is a contradiction. □

Thus, there are conditions under which separating is possible in the principal-worker model but not in the two principal one worker model. This is not a formal proof for the statement that the separating equilibrium is harder to obtain when the market for labour is competitive (since we would have to show that there are no alternate conditions under which separation is possible when the market for labour is competitive but not in the principal agent model) . However, given that the intuition for the separating equilibrium is exactly the same in both environments, it is easy to check that the desired result follows directly from proposition 4.

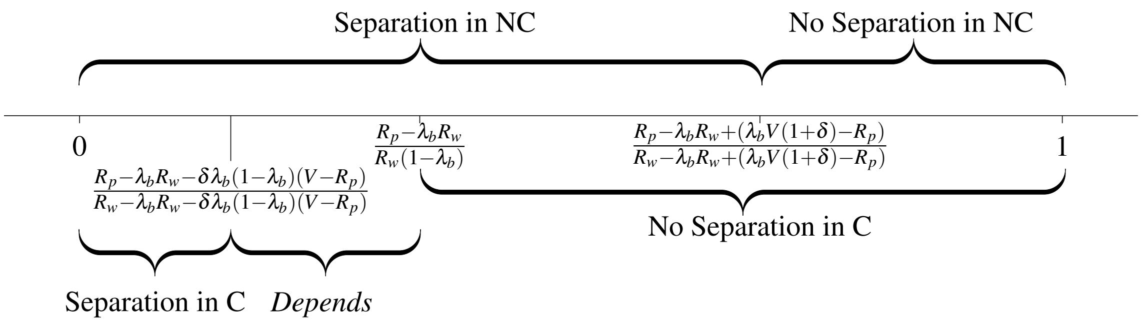
The following is a pictorial representation of the comparison between the principal-worker model with the model in which two principals compete for the same worker (when condition required for uniqueness of separating equilibrium in the principal agent model holds). The picture highlights when separating equilibria are possible. The labels in the figure have the following meaning:

Labels

C = Competition for labour i.e. the two Principal- one worker model

NC = No Competition for labour i.e. the Principal-worker model

Figure 1: Prior Reputation (p_g) on x axis



5 Pooling equilibrium in the baseline model

In this section, I am interested in understanding when worker strategies can lead to a pooling equilibrium where both type workers choose to leave to form their own firm. I show that under conditions similar to those needed to get a separating equilibrium in proposition 1 (from the main body of this paper), then there cannot exist a pooling equilibrium. For this section, I will assume that: $V - R_w + \delta(\lambda_b V + (1-\lambda_b)\lambda_b V) > 0$. This condition requires that the maximum payoff that a B type worker may get from a pooling equilibrium be positive. If this condition is not met then it is not IR for the B worker to form the firm in period one. Therefore, without this condition, there does not exist any pooling PBE of the game. Also, I will assume that $V < R_p$. This stops the principal from forming the firm if there one period gains only. This assumption simplifies second period analysis. Mistake probabilities are the same for G and B type workers. Also, we will need that the prior reputation is high enough to have a chance of getting pooling on L equilibrium in period 1. This is formalized in the following claim:

Claim 6. *There exists a p' such that if $p_g \in [0, p']$, there does not exist a pooling equilibrium.*

Proof. We know that:

$\lambda_b V - R_w + \delta(\lambda_b V) < 0$ and $V - R_w + \delta(\lambda_b V + (1-\lambda_b)\lambda_b V) > 0$. By intermediate value theorem there exists a p' such that it is IR for B worker to play L in period 1 as part of a pooling equilibrium only if $p_g > p'$:

$$p_g V + (1-p_g)\lambda_b V - R_w + \delta(\lambda_b[\frac{p_g V + (1-p_g)\lambda_b \lambda_b V}{p_g + (1-p_g)\lambda_b}] + (1-\lambda_b)\lambda_b V) > 0 \Leftrightarrow p_g > p'$$

To find value of p':

$$p_gV+(1-p_g)\lambda_bV-R_w+\delta[\lambda_b(\frac{p_g}{p_g+(1-p_g)\lambda_b}V+\frac{(1-p_g)\lambda_b}{p_g+(1-p_g)\lambda_b}\lambda_bV)+(1-\lambda_b)\lambda_bV]>0$$

$$Let\ x=p_g+(1-p_g)\lambda_b$$

$$Then\ above\ reduces\ to:$$

$$Vx^2+x(\delta\lambda_bV(1-\lambda_b)-R_w)+\delta\lambda_bV(p_g+(1-p_g){\lambda_b}^2)>0$$

$$Now\ p_g+(1-p_g){\lambda_b}^2=x\lambda_b+p_g(1-\lambda_b)$$

$$And\ p_g(1-\lambda_b)=x-\lambda_b\Rightarrow\ p_g+(1-p_g){\lambda_b}^2=x\lambda_b+x-\lambda_b$$

$$Therefore\ we\ have$$

$$x^2V+x(\delta\lambda_bV(1-\lambda_b)-R_w)+\delta\lambda_bV(x(1+\lambda_b)-\lambda_b)>0$$

$$\Leftrightarrow x^2V+x(2\delta\lambda_bV-R_w)-\delta{\lambda_b}^2V>0$$

$$\Leftrightarrow (x\sqrt{V}+\frac{2\delta\lambda_bV-R_w}{2\sqrt{V}})^2-(\sqrt{(\frac{2\delta\lambda_bV-R_w}{2\sqrt{V}})^2+\delta{\lambda_b}^2V})^2>0$$

$$\Leftrightarrow x>\frac{\sqrt{(\frac{2\delta\lambda_bV-R_w}{2\sqrt{V}})^2+\delta{\lambda_b}^2V}+\frac{R_w-2\delta\lambda_bV}{2\sqrt{V}}}{\sqrt{V}}$$

$$\Leftrightarrow p_g>\frac{\sqrt{(\frac{2\delta\lambda_bV-R_w}{2\sqrt{V}})^2+\delta{\lambda_b}^2V}+\frac{R_w-2\delta\lambda_bV}{2\sqrt{V}}-\lambda_b\sqrt{V}}{\sqrt{V}(1-\lambda_b)}$$

We can show that the RHS is above zero and less than one. The former uses $R_w>\lambda_bV(1+\delta)$ (assumed in the model) and the latter uses the condition we assumed in the beginning of the subsection $V-R_w+\delta(\lambda_bV+(1-\lambda_b)\lambda_bV)>0$.

Therefore, the exact expression for p' is $p'=(\sqrt{(\frac{2\delta\lambda_bV-R_w}{2\sqrt{V}})^2+\delta{\lambda_b}^2V}+\frac{R_w-2\delta\lambda_bV}{2\sqrt{V}}-\lambda_b\sqrt{V})/(\sqrt{V}(1-\lambda_b))$

□

For the rest of this subsection, I will assume that $p_g>p'$.

Proposition 5. *If $p_g\geq max\{p',\frac{R_p-\lambda_bR_w}{R_w-\lambda_bR_w}\}$, then there does not exist a PBE pooling equilibrium where both types of workers choose to play L in period 1.*

Proof. Suppose there is a pooling on L equilibrium. Then, by assumption $V<R_p$, in a pooling equilibrium

where the worker forms his firm in period 1, the principal gets a payoff of zero. Therefore, it must be the case that to any contract that the principal offers, either the optimal response for the G,B worker is to play L or the optimal response is such that it gives the principal a negative payoff (not IR for principal).

I will show that there is a deviation for the principal, where, if the two types of workers respond optimally, the principal gets positive payoff. Consider the contract $\{V - R_w + \delta V, 0\}$. Since this is the best payoff a G player can achieve by L, he will accept it. Given the contract and G worker's best response, it is optimal for the B worker to accept it too. Payoff to principal in this case is positive if:

$$\begin{aligned}
&p_g V + (1-p_g)\lambda_b V - (p_g(V - R_w + \delta V) + (1-p_g)\lambda_b(V - R_w + \delta V)) - R_p + \\
&\delta[p_g(\frac{p_g}{p_g + (1-p_g)\lambda_b}V + \frac{(1-p_g)\lambda_b}{p_g + (1-p_g)\lambda_b}\lambda_b V) + (1-p_g)(\lambda_b(\frac{p_g}{p_g + (1-p_g)\lambda_b}V + \frac{(1-p_g)\lambda_b}{p_g + (1-p_g)\lambda_b}\lambda_b V) + (1-\lambda_b)\lambda_b V)] > 0 \\
&\Leftrightarrow p_g > \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}
\end{aligned}$$

Now $p_g \geq max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\}$. Therefore, principal has a deviation which gives him positive payoffs i.e. a higher payoff than what he would receive in a pooling on L equilibrium. Thus, the principal will deviate and so there is no pooling on L equilibrium if $p_g \geq (R_p - \lambda_b R_w)/(R_w - \lambda_b R_w)$. □

Corollary 6. *Let $max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\} = \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}$.*

A necessary condition for pooling on L PBE to exist is:

$$p' \leq p_g \leq \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}.$$

If $max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\} = p'$ then there is no pooling on L equilibrium.

Suppose the above holds. Note that if we pick a $p_g \in [0, p']$, then we will have separating equilibrium[15] (by proposition 1 in main body of this paper) but we will not have a pooling equilibrium.

A Appendix

A.1 Competitive Labour market with types being Common knowledge

When two principals compete for one worker, the worker gets offered high wage contracts and competition between principals erode their payoff to zero. In this subsection, I establish these results in the baseline case where worker type is known. Once again, I look at possible equilibria in period 2 first and then discuss equilibria of the entire game.

[15] Since $(R_p - \lambda_b R_w)/(R_w - \lambda_b R_w) < (R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))/(R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))$, $p' < (R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))/(R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))$.

A.1.1 Period 2

Claim 7. *If the worker had already formed his firm in period 1 then the principals cannot offer a contract which is IR for the principal to offer and which the worker will accept in period 2.*

Proof. If the worker had already formed his firm in period one and his type is known then he expects to get his expected product in period 2 by choosing to stay with his own firm i.e. G worker gets price V and B worker will get price $\lambda_b V$. Whichever firm the worker works with the customers will pay these prices. Neither principal can afford to pay the worker all of what they receive from the customer since they have a positive cost of firm formation. □

Corollary 7. *Suppose the worker formed his own firm in period 1.If the worker type is G, then the principals will offer a zero wage contract in period 2 which will only be accepted upon a mistake. If the worker type is B, the principals will offer no contracts*

Proof. Follows from $V > R_p$ and $\lambda_b V < R_p$ □

Claim 8. *Suppose the worker did not form his own firm in period one. Suppose the worker accepted a contract with principal i in period 1. If the worker is of type G, then in period 2, principal i will offer contracts of the form $\{x,y\}$ where the probability of offering a contract with $x \leq x_0$ is given by $F_i(x_0) = \frac{Ax_0}{V-R_p-x_0}$; $A = \frac{\alpha\varepsilon}{3-4\alpha\varepsilon}$ and the support of the distribution is $x \in (0,(V-R_p)\frac{3-4\alpha\varepsilon}{3(1-\alpha\varepsilon)}]$, $y \in \mathbb{R}^+$. Principal $j(\neq i)$ will offer contracts of the form $\{a,b\}$ where the probability of offering a contract with $a \leq a_0$ is given by $F_j(a_0) = \frac{Ba_0}{V-a_0}$; $B = \frac{R_p}{V} + A$ and the support of this distribution is $a \in [0,(V-R_p)\frac{3-4\alpha\varepsilon}{3(1-\alpha\varepsilon)}], b \in \mathbb{R}^+$ with a mass point at $a = 0$ where the mass at zero is $\frac{R_p}{V}$.*

Proof. It is clear that the principal who signs the worker in period one has a strong advantage in period 2. This is because that principal has already invested R_p in period 2. So, while principal j can offer a maximum wage of $V-R_p$ in period 2 for a G worker, principal i can offer a higher wage. This is why the payoff for the principals according to the strategy above is (as $\varepsilon \to 0$) R_p for principal i, 0 for principal j and $V-R_p$ for the good type worker. The proof is similar to the proof in claim 11 □

Claim 9. *Suppose the worker did not form his own firm in period one. Suppose the worker accepted a contract with principal i in period 1. If the worker is of type B, then in period 2, principal i will offer the contract $\{0,0\}$ and the other principal will offer no contract.*

Proof. Follows from $R_p > \lambda_b V$. □

Corollary 8. *If the type B worker accepted a contract with principal i in period 1, then in any PBE, the expected payoffs for the B worker is* 0 *in period 2, the expected payoffs for principal i is approximately*[16] $\lambda_b V$ *and the expected payoff for principal* $j (j \neq i)$ *is approximately* 0.

Claim 10. *Suppose the worker did not form his own firm in period one. Suppose the worker played N in period 1. If the worker is of type B, then in period 2, both principals will offer no contract.*

Proof. Follows from $R_p > \lambda_b V$. □

Claim 11. *Suppose the worker did not form his own firm in period one. Suppose the worker played N in period 1. If the worker is of type G, then in period 2, the two principals will play a symmetric mixed strategy where they will offer contracts paying less than* $\{x, 0\}$ *with probability* $\frac{Ax}{V - R_p - x}$ *where* $A = \frac{\alpha\varepsilon}{3 - 4\alpha\varepsilon}$ *and support of the distribution is* $x \in [0, \frac{(V-R_p)(3-4\alpha\varepsilon)}{3(1-\alpha\varepsilon)}]$.

Proof. It is easy to show that there is no pure strategy equilibrium for the game between the two principals. I will confine my investigation into symmetric mixed strategy equilibrium into contracts of the form $\{y, 0\}$. Since the G type worker never fails, any contract which has a positive second contract is equivalent to a contract with zero wages for failure. Suppose principal 1 chooses the contract $\{x, 0\}$ and principal 2 chooses the contract $\{y, 0\}$. Then utility function for principal 1 is given by:

$$\begin{aligned} u(x,y) &= (1-\alpha\varepsilon)(V - R_p - x) \ ; \ x > y \\ &= \frac{1-\alpha\varepsilon}{2}(V - R_p - x) \ ; \ x = y \\ &= \frac{\alpha\varepsilon}{3}(V - R_p - x) \ ; \ x < y \end{aligned}$$

If both principals are mixing according to the same distribution F then expected utility from $\{x, 0\}$:

$$F(x)(1-\alpha\varepsilon)(V - R_p - x) + (1 - F(x))\frac{\alpha\varepsilon}{3}(V - R_p - x)$$

The first order condition to the above equation gives us a differential equation the solution to which (using boundary condition $F(0) = 0$) reveals $F(x) = (Ax)/(V - R_p - x)$ where $A = (\alpha\varepsilon)/(3 - 4\alpha\varepsilon)$ □

Thus payoffs to the players in period 2 are given by the following matrices. The first coordinate is the payoff to principal 1, second is payoff to principal 2, third is worker. The payoffs are limits of when $\varepsilon \to 0$.

[16] as $\varepsilon \to 0$

Table 7: Payoffs in Period 2

Table 8: Payoff if principal 1's contract accepted in Period 1

$(P1,P2,G)$	$(P1,P2,B)$
$(R_p,0,V-R_p)$	$(\lambda_b V,0,0)$

Table 9: Payoff if Spinoff in Period 1

$(P1,P2,G)$	$(P1,P2,B)$
$(0,0,V)$	$(0,0,\lambda_b V)$

Table 10: Payoff if N in Period 1

$(P1,P2,G)$	$(P1,P2,B)$
$(0,0,V-R_p)$	$(0,0,0)$

A.1.2 Period 1

Suppose the worker is of type G.

Suppose the two principals offer the contracts $\{x,0\}$,$\{y,0\}$ in period 1. Then payoff to G worker from his action choice accept $\{x,0\}$ is:

$$\begin{aligned}\{x,0\} \rightarrow & (1-\alpha\varepsilon)(x+\delta[(1-\alpha\varepsilon)E(max\{F1,F2\})+\frac{\alpha\varepsilon}{3}E(min\{F1,F2\})+\frac{\alpha\varepsilon}{3}(V-R_w)+\frac{\alpha\varepsilon}{3}.0])\\ &+\frac{\alpha\varepsilon}{3}(y+\delta[(1-\alpha\varepsilon)E(max\{F1,F2\})+\frac{\alpha\varepsilon}{3}E(min\{F1,F2\})+\frac{\alpha\varepsilon}{3}(V-R_w)+\frac{\alpha\varepsilon}{3}.0])\\ &+\frac{\alpha\varepsilon}{3}(V-R_w+\delta(1-\alpha\varepsilon)V)\\ &+\frac{\alpha\varepsilon}{3}(0+\delta[(1-\alpha\varepsilon)E(max\{F,F\})+\frac{\alpha\varepsilon}{3}E(min\{F,F\})+\frac{\alpha\varepsilon}{3}(V-R_w)+\frac{\alpha\varepsilon}{3}.0])\end{aligned}$$

Similarly expected payoff from accepting $\{y,0\}$. Payoff from leaving and forming own firm is given by:

$$\begin{aligned}L \rightarrow & (1-\alpha\varepsilon)(V-R_w+\delta(1-\alpha\varepsilon)V)\\ &+\frac{\alpha\varepsilon}{3}(x+\delta[(1-\alpha\varepsilon)E(max\{F1,F2\})+\frac{\alpha\varepsilon}{3}E(min\{F1,F2\})+\frac{\alpha\varepsilon}{3}(V-R_w)+\frac{\alpha\varepsilon}{3}.0])\\ &+\frac{\alpha\varepsilon}{3}(y+\delta[(1-\alpha\varepsilon)E(max\{F1,F2\})+\frac{\alpha\varepsilon}{3}E(min\{F1,F2\})+\frac{\alpha\varepsilon}{3}(V-R_w)+\frac{\alpha\varepsilon}{3}.0])\\ &+\frac{\alpha\varepsilon}{3}(0+\delta[(1-\alpha\varepsilon)E(max\{F,F\})+\frac{\alpha\varepsilon}{3}E(min\{F,F\})+\frac{\alpha\varepsilon}{3}(V-R_w)+\frac{\alpha\varepsilon}{3}.0])\end{aligned}$$

Since we are concerned about the question - when can workers leave to form their own firm, I will assume that $V-R_w+\delta V > \delta(V-R_p)$. Else, no worker will ever choose the action of forming own firm. In any equilibrium, the worker's action choice is clear. He looks at the two contracts on offer and chooses between the best contract and the option L[17]. Consider the game between the principals. For principal 1,

[17] Option to play N is dominated by option to form own firm.

worker will not choose contract 1 unless :

$$
\begin{aligned}
&payoff\ from\ \{x,0\}\ \geq\ payoff\ from\ L \\
\Leftrightarrow & x \geq V - R_w + \delta[(1-\alpha\varepsilon)(V - E(max\{F1,F2\})) - \frac{\alpha\varepsilon}{3}E(min\{F1,F2\}) - \frac{\alpha\varepsilon}{3}(V - R_w)]
\end{aligned}
$$

Let $C^* = V - R_w + \delta[(1-\alpha\varepsilon)(V - E(max\{F1,F2\})) - \frac{\alpha\varepsilon}{3}E(min\{F1,F2\}) - \frac{\alpha\varepsilon}{3}(V - R_w)]$.

The utility function function of the principal offering contract $\{x,0\}$ if the other principal is offering $\{y,0\}$ is given by:

$$
\begin{aligned}
u(x,y) &= \frac{\alpha\varepsilon}{3}[P1_{win}] + (1-\alpha\varepsilon)(w_L) + \frac{\alpha\varepsilon}{3}[P2_{win}] + \frac{\alpha\varepsilon}{3}(w_N)\ ;\ x<y, y<C^* \\
&= \frac{\alpha\varepsilon}{3}[P1_{win}] + (1-\alpha\varepsilon)(P2_{win}) + \frac{\alpha\varepsilon}{3}[w_L] + \frac{\alpha\varepsilon}{3}(w_N)\ ;\ x<y, y\geq C^* \\
&= \frac{\alpha\varepsilon}{3}[P1_{win}] + (1-\alpha\varepsilon)(w_L) + \frac{\alpha\varepsilon}{3}[P2_{win}] + \frac{\alpha\varepsilon}{3}(w_N)\ ;\ x>y, x<C^* \\
&= \frac{\alpha\varepsilon}{3}[w_L] + (1-\alpha\varepsilon)(P1_{win}) + \frac{\alpha\varepsilon}{3}[P2_{win}] + \frac{\alpha\varepsilon}{3}(w_N)\ ;\ x>y, x\geq C^*
\end{aligned}
$$

$$
\begin{aligned}
&where: \\
&P1_{win} = V - R_p - x + \delta[\int f_1(x)((F_2(x)+b)(1-\alpha\varepsilon)(V-x) + (1-F_2(x)-b)\frac{\alpha\varepsilon}{3}(V-x))dx] \\
&w_L = 0 + \delta[\frac{\alpha\varepsilon}{3}(V - R_p)] \\
&P2_{win} = 0 + \delta[\int f_2(x)((F_1(x))(1-\alpha\varepsilon)(V-R_p-x) + (1-F_1(x))\frac{\alpha\varepsilon}{3}(V-R_p-x))dx + \frac{R_p}{V}\frac{\alpha\varepsilon}{3}(V-R_p)] \\
&w_N = 0 + \delta[\int f(x)((F(x))(1-\alpha\varepsilon)(V-R_p-x) + (1-F(x))\frac{\alpha\varepsilon}{3}(V-R_p-x))dx]
\end{aligned}
$$

It is clear that when $\varepsilon \approx 0$, the best payoff is from $x > y, x \geq C^*$ and that there is no pure strategy equilibrium in the game between the principals.

Proposition 6. *If the worker type is known and the worker is of type G then both principals play the following symmetric mixed strategy in period 1: principals bid a contract which pays less than $\{x,0\}$ with probability $G(x)$ where:*

$$
G(x) = \frac{Ax}{P1_{win} - P2_{win}}\ ;\ A = \frac{\alpha\varepsilon}{3-4\alpha\varepsilon}\ ,\ x \in [C^*, d]
$$

where d is such that:

$$
\begin{aligned}
&\frac{\alpha\varepsilon}{3}[V - R_p - C^* + \delta[\int f_1(x)((F_2(x)+b)(1-\alpha\varepsilon)(V-x) + (1-F_2(x)-b)\frac{\alpha\varepsilon}{3}(V-x))dx]] + (1-\alpha\varepsilon)(P2_{win}) + \frac{\alpha\varepsilon}{3}[w_L] + \frac{\alpha\varepsilon}{3}(w_N) \\
&= \\
&\frac{\alpha\varepsilon}{3}[w_L] + (1-\alpha\varepsilon)(V - R_p - d + \delta[\int f_1(x)((F_2(x)+b)(1-\alpha\varepsilon)(V-x) + (1-F_2(x)-b)\frac{\alpha\varepsilon}{3}(V-x))dx]) + \frac{\alpha\varepsilon}{3}[P2_{win}] + \frac{\alpha\varepsilon}{3}(w_N)
\end{aligned}
$$

The G type worker accepts the better of the two realizations of contracts offered from the above distribution.

Proof. Very similar to proof for claim 11. To show that worker will accepts contract it is sufficient to show that worker accepts the lowest contract of $\{C^*, 0\}$. This is easy to check. $\square$

Thus, if the worker type is G, then in the full information case the worker will not form his own firm and the expected payoff to the worker will be approximately $V - R_p + \delta V$. The principals get an expected payoff of zero. These payoffs have been calculated as $\varepsilon \to 0$.

Similarly it can be shown that if the worker type is B, then in the full information case, the worker will not form his own firm. The payoff to the worker will be $\lambda_b V - R_p + \delta \lambda_b V$ and competition between the principals will force their expected payoff to zero.

Thus payoffs in the equilibrium in the full information case can be summarized as follows. First coordinate is payoff for principal 1, second is payoff for principal 2, and third is payoff for worker.

Table 11: Expected Payoffs if Types are Known and Competitive Labour Market

$(Principal\ 1, Principal\ 2, G)$	$(Principal\ 1, Principal\ 2, B)$
$(0, 0, V - R_p + \delta V)$	$(0, 0, \lambda_b V(1+\delta) - R_p)$

A.2 Competitive Labour Market and private types

A.2.1 Period 2

Once again, I will solve the one period game beginning in period 2 first and then proceed by backwards induction. The relevant state variables which affect decisions in period 2 are:

1. Worker formed own firm in period 1 (f), did not form and accepted contract (nf) or did nothing N.
2. If worker played nf in period one, then which firm's contract did it accept ($P1$ or $P2$).[18] $P1$ stands for principal one.
3. Reputation of worker at the end of period 1. Let this be denoted by p_g^2.

In equilibrium, strategies in period 2, therefore are functions from the state variables augmented history. In particular, the strategy for principal i ($i \in \{1, 2\}$) in period 2 depends only on the value taken by the state

[18]This is relevant because the principal of that firm has already invested R_p in forming the firm which the other principal has not.

variables:

$$s^i_{p,2} : (nf, \{P1, P2\}, p^2_g) \rightarrow \{x \in 2^{\{(s,f);\ s,f \geq 0\}};\ 0 < |x| < 3\}$$

$$s^i_{p,2} : (f/N, p^2_g) \rightarrow \{x \in 2^{\{(s,f);\ s,f \geq 0\}};\ 0 < |x| < 3\}$$

So the strategy for principals in period two depend only upon reputation of worker at the end of period one if the worker had already formed his own firm in period one or played N. If the worker accepted a contract in period one, then the strategy for principal i also depends upon whether the worker had formed a firm with him in period one or the other principal. This is because the principal who formed the firm in period one has already invested R_p and is thus able to offer different contracts from the principal who did not form a firm in period one.

In case the worker did not form his firm in period one, then his decision in period two does not depend upon the particular principal he chose to form the firm with in period one or if he played N. Therefore, strategy for worker in period 2 depends upon the whether he formed his own firm in period one or not, reputation, actions taken by the principals in period 2 and own type. Let $\{\{s^2_1, f^2_1\}_i, \{s^2_2, f^2_2\}_i\}$ be the contracts offered by principal i in period 2. Let acc^i_j means accept contract j offered by principal i.

$$s_{w,2} : (nf/N, p^2_g, \{\{s^2_1, f^2_1\}_1, \{s^2_2, f^2_2\}_1\}, \{\{s^2_1, f^2_1\}_2, \{s^2_2, f^2_2\}_2\}, T) \rightarrow \{N, acc^1_1, acc^1_2, acc^2_1, acc^2_2, L\}\ ;\ T \in \{G, B\}$$

$$s_{w,2} : (f, p^2_g, \{\{s^2_1, f^2_1\}_1, \{s^2_2, f^2_2\}_1\}, \{\{s^2_1, f^2_1\}_2, \{s^2_2, f^2_2\}_2\}, T) \rightarrow \{N, acc^1_1, acc^1_2, acc^2_1, acc^2_2, S\}\ ;\ T \in \{G, B\}$$

Strategy for customers in period 2:

$$s_{c,2} : (f/nf/N, p^2_g, \{\{s^2_1, f^2_1\}_1, \{s^2_2, f^2_2\}_1\}, \{\{s^2_1, f^2_1\}_2, \{s^2_2, f^2_2\}_2\}, a_w(2)) \rightarrow \{b\ ;\ b \in \mathbb{R}_+\}$$

Case 1 - Worker played f in period 1

If the worker failed, then the only equilibrium is one where the B type worker (for a worker who fails must be B type) plays S and both principals offer no contracts. This follows from the assumption that $R_p > \lambda_b V$.

Suppose the worker succeeded in period 1. We will be interested in the equilibria where p^2_g is high. This is because we are interested in separating equilibria in which G worker leaves to form firm. In such equilibria

p_g^2 will be close to 1.

Assume $p_g^2 \approx 1$.

Pooling on S Equilibrium

Consider the following strategies in period 2:

For Principals (both):

$s_p(f, p_g^2 \approx 1) = \{\{0,0\}\}$

For Worker:

$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1 \text{ ; } s_1^1 \geq s_2^1 \text{ and } s_1^1 > p_g^2 V + (1 - p_g^2)\lambda_b V$

$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_2 \text{ ; } s_2^1 \geq s_1^1 \text{ and } s_2^1 > p_g^2 V + (1 - p_g^2)\lambda_b V$

$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = S \text{ ; Else.}$

$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_1 \text{ ; } \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 \text{ and } \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > p_g^2 V + (1 - p_g^2)\lambda_b V$

$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_2 \text{ ; } \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 > \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 \text{ and } \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 > p_g^2 V + (1 - p_g^2)\lambda_b V$

$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = S \text{ ; Else.}$

Proof. Essentially, if p_g^2 is high enough then the payoff from pooling on S in period 2 is above $V - R_p$. Since neither principal can offer more, clearly there will be a pooling on S equilibrium. □

Payoffs as $\varepsilon \to 0$:

$$P1 \to 0$$

$$P2 \to 0$$

$$G \to p_g^2 V + (1 - p_g^2)\lambda_b V$$

$$B \to p_g^2 V + (1 - p_g^2)\lambda_b V$$

If assumption A holds, then above is the only equilibrium after worker forms his own firm. If assumption A does not hold then the above remains an equilibrium but the following equilibria are also possible. I describe the outcome in these equilibria next. The proofs are similar to the proof of claim 11.

Other Equilibria possible in period 2

Here, I don't describe other equilibrium play in period 2 in any detail. I only mention the play in equilibrium and the equilibrium payoff only. The proof is deliberately left out as it is a similar (to the work above) exercise in constructing the right strategies to justify the play below.

1. There is an equilibrium where both principals offer the following contract

$$\{\frac{p_g^2(1-\alpha\varepsilon)V+\frac{\alpha\varepsilon}{3}\lambda_b V(1-p_g^2)}{p_g^2(1-\alpha\varepsilon)+\frac{\alpha\varepsilon}{3}\lambda_b(1-p_g^2)}-R_p-\frac{\alpha\varepsilon}{3}\frac{p_g^2V+(1-p_g^2)\lambda_b V-R_p}{p_g^2\frac{1-\alpha\varepsilon}{2}+\frac{\alpha\varepsilon}{3}\lambda_b(1-p_g^2)},0\} \tag{4}$$

   Worker type G accepts one of the contracts at random and worker type B plays S.

   Payoffs as $\varepsilon \to 0$:

$$P1 \to 0$$
$$P2 \to 0$$
$$G \to V-R_p$$
$$B \to \lambda_b V$$

2. There is an equilibrium where both principals offer the following contract

$$\{\frac{p_g^2(1-\alpha\varepsilon)V+\frac{\alpha\varepsilon}{3}\lambda_b V(1-p_g^2)}{p_g^2(1-\alpha\varepsilon)+\frac{\alpha\varepsilon}{3}\lambda_b(1-p_g^2)}-R_p-\frac{\alpha\varepsilon}{3}\frac{p_g^2V+(1-p_g^2)\lambda_b V-R_p}{p_g^2\frac{1-\alpha\varepsilon}{2}+\frac{\alpha\varepsilon}{3}\lambda_b(1-p_g^2)},0\} \tag{5}$$

   Worker type G, randomizes between accepting a contract and playing S. Worker B plays S.

   Payoffs as $\varepsilon \to 0$:

$$P1 \to 0$$
$$P2 \to 0$$
$$G \to V-R_p$$
$$B \to V-R_p$$

3. There is an equilibrium where both principals offer the following contract

$$\{\lambda_b V, 0\} \tag{6}$$

   Worker type G, accepts contract. Worker B plays S. Worker type G does not accept any other IR contract.

Payoffs as $\varepsilon \to 0$:

$$P1 \to \frac{1}{2} p_g^2 (V - \lambda_b V - R_p)$$
$$P2 \to \frac{1}{2} p_g^2 (V - \lambda_b V - R_p)$$
$$G \to \lambda_b V$$
$$B \to \lambda_b V$$

In general, every time I show the existence of a separating equilibrium, I will make use of the pooling on S equilibrium in period 2.

Case 2 - Worker played nf in period 1 and accepted principal 1's contract

In this case, the worker strategies in equilibrium will always be - accept the highest paying contract. This is because leaving and forming own firm is no longer individually rational.

Subcase 1 - $p_g^2 < (R_p - \lambda_b V)/(V - \lambda_b V)$

In this case, we have that $p_g^2 V + (1 - p_g^2)\lambda_b V - R_p < 0$.

Equilibrium 1

Principal 1 offers a zero wage contract. Principal 2 offers no contract. Worker accepts principal 1's contract. The fact that these strategies constitute an equilibrium follows from $p_g^2 V + (1 - p_g^2)\lambda_b V - R_p < 0$ and equal probability of mistake for the two types of workers.

Payoffs as $\varepsilon \to 0$:

$$P1 = p_g^2 V + (1 - p_g^2)\lambda_b V$$
$$P2 = 0$$
$$G = 0$$
$$B = 0$$

Proposition 7. *No other equilibrium outcomes in this case.*

Proof. It is not incentive compatible for the the worker to form his own firm. Therefore the only other possible equilibrium outcome occurs if principal 2 offers a contract and atleast one type of worker accepts this contract. Since $p_g^2 V + (1 - p_g^2)\lambda_b V - R_p < 0$, it will only be incentive compatible for principal 2 to offer a contract if the G, B worker's strategies are such that the G worker is more likely to accept the contract than the B worker. In this case, it is always possible and incentive compatible for principal 1 to outbid principal 2.

This is because principal 2 has to put in the extra cost of firm formation at period 2 whereas principal 1 is not burdened with this cost any more (since he already made the fixed investment at period 1). Thus, principal 1 will always outbid principal 2. The following highlights the four cases in which the good worker may accept principal 2's contract with higher probability. I rule them out.

Case 1 - G worker accepts contract with principal 2 with probability 1 and B worker accepts a contract with principal 1

Suppose the contract accepted by the G worker is $\{x,y\}$. Principal 1 will offer a contract to make the B worker just indifferent between the two contracts. This implies that the expected payoff for principal 1 is $(1-p_g^2)(\lambda_b V - \lambda_b x - (1-\lambda_b)y))$. We can easily check that if principal 1 deviates to $\{x+\mu,0\}$ where μ is an extremely small positive real number, then principal gets a higher payoff. So this cannot be an equilibrium.

Case 2 - G worker accepts contract with principal 2 with probability 1 and B worker accepts a contract with principal 1 with positive probability

Same argument as above.

Case 3 - G worker accepts contract with principal 2 with positive probability and B worker accepts a contract with principal 2 with a lower positive probability

This can only happen if both principals offer the same contract. Principal 1 can always deviate by offering a little higher reward for success.

Case 4 - G worker accepts contract with principal 2 with positive probability and B worker accepts a contract with principal 1 probability 1

Suppose the contract accepted by the G worker is $\{x,y\}$. Principal 1 will have to offer x for success since the good worker is indifferent. Since the bad type is choosing principal 1's contract principal one must be offering a contract $\{x,y'\}$ where $y' \geq y$. Moreover, for this to be an equilibrium $x = V - R_p$. This is to prevent principal 2 from deviating and offering a little more for success and getting the good worker for sure. Also, in equilibrium, there will be no need for principal 1 to pay anything strictly above y for failure. So $y = y'$. We can easily check that if principal 1 deviates to $\{x+\mu,y\}$ where μ is an extremely small positive real number, then principal 1 gets a higher payoff. So this cannot be an equilibrium.

$\square$

Subcase 2 - $p_g^2 > (R_p - \lambda_b V)/(V - \lambda_b V)$

above implies that $p_g^2 V + (1-p_g^2)\lambda_b V > R_p$. Suppose $\varepsilon \to 0$.

Equilibrium 1 -

P1, P2 both don't offer any reward for failure and they randomize among contracts over payment for success. P1 offers contracts of the form $\{x,0\}$ where $x \in (0,a]$ and P2 offers contracts of the form $\{y,0\}$ where $y \in [0,a)$. $a = ((1-(4\alpha\varepsilon)/(3))(V(p^2+(1-p_g^2)\lambda_b)-R_p))/((1-\alpha\varepsilon)(p^2+(1-p_g^2)\lambda_b))$. P2 has a mass point of $\frac{R_p}{p^2V+(1-p_g^2)\lambda_b V}$ on the contract $\{0,0\}$. P2 mixes with the distribution F_2 where probability that P2 offers less than y for success is given by:

$$F_2(y) = \frac{A_2 y}{(V-y)(p^2+(1-p_g^2)\lambda_b)(1-\frac{4\alpha\varepsilon}{3})}$$
$$;\ where\ A_2 = \frac{R_p}{V}(1-\frac{4\alpha\varepsilon}{3}) + \frac{\alpha\varepsilon}{3}(p_g^2+(1-p_g^2)\lambda_b)$$

P1 mixes on with the distribution F_1 where probability that P1 offers less than x for success is given by

$$F_1(x) = \frac{A_1 y}{(1-\frac{4\alpha\varepsilon}{3})((V-y)(p^2+(1-p_g^2)\lambda_b)-R_p)}$$
$$;\ where\ A_1 = \frac{\alpha\varepsilon}{3}(p^2+(1-p_g^2)\lambda_b)$$

G,B type worker accept whichever contract gives them highest payoff.

Proof. Similar to claim 11. □

Payoffs as $\varepsilon \to 0$

$$P1 = R_p$$
$$P2 = 0$$
$$G = V - \frac{R_p}{p_g^2+(1-p_g^2)\lambda_b}$$
$$B = \lambda_b(V - \frac{R_p}{p_g^2+(1-p_g^2)\lambda_b})$$

Equilibrium 2 - P1 offers contracts of the form $\{x,f(x)\}$ where $x \in (0,a]$ and P2 offers contracts of the form $\{y,0\}$ where $y \in [0,a)$. $f(x)$ is such that $\lambda_b x + (1-\lambda_b)f(x) = \lambda_b a$. So P1 always gets B worker. $a = V - R_p$ as $\varepsilon \to 0$. P2 has a mass point of (R_p/V) on the contract $\{0,0\}$. P1, P2 mix on payoff fo success with distributions G_1, G_2.

Proof. Similar to claim 11. □

Payoffs as $\varepsilon \to 0$

$$P1 = R_p(p_g^2 + (1-p_g^2)\lambda_b)$$

$$P2 = 0$$

$$G = V - R_p$$

$$B = \lambda_b(V - R_p)$$

Lemma 1. *Let history be nf, acc_1, p_g^2. As $\varepsilon \to 0$, P2's payoff in any equilibrium in period 2 goes to zero.*

Proof. Suppose $P2$ gets positive payoff in some equilibrium in period 2. It is easy to show that any equilibrium must be in mixed strategies. We can show that $P2$ will never pay for failure in any equilibrium. Now let the range of payoffs for success be (a,b). If $P2$ gets positive payoff in equilibrium, then $P2$ must get positive payoff at all points in its support. Payoff of $P2$ from $\{a,o\}$ is positive implies $P1$ must have a mass point at $\{a,f\}$ for some f. Let f' be the highest value such that P1 has a non atomic mass on $\{a,f'\}$.

Case 1 - $\lambda_b a + (1-\lambda_b)f \geq \lambda_b b$

P2 cannot have a mass point at $\{b,0\}$ else P1 could pay a little more and increase payoff. If $P2$ does not have a mass point at $\{b,0\}$ then $\lambda_b a + (1-\lambda_b)f \geq \lambda_b b \Rightarrow$ P1 always gets B worker if he offers $\{a,f\}$.

Also, P2 cannot have a mass point at $\{a,0\}$ else P1 can increase payoff by shifting mass point to the right. P2 does not have mass point at $a,0$ implies that $P1$ always loses G type when he offers $\{a,f\}$.

Now compare payoffs for P1 at $\{b,0\}$ and $\{a,f\}$. They must be the same but they are not. Contradiction.

Case 2 - $\lambda_b a + (1-\lambda_b)f < \lambda_b b$

Then there exists $x \in (a,b)$ such that $\lambda_b a + (1-\lambda_b)f < \lambda_b x$. P2 must be indifferent between $\{x-\nu,0\}$ and $\{x+\nu,0\}$ for all ν. But if P1 has a mass point at $\{a,f\}$, then this is not possible. Contradiction.

Therefore P1 cannot have a mass point at $\{a,f\}$. Contradiction.

Thus $P2$ gets zero payoff in all equilibria in period 2.

□

Claim 12. *In any equilibrium in period 2, G worker cannot get more than $V - R_p$ and G worker cannot get less than $V - (R_p)/(p_g^2 + (1-p_g^2)\lambda_b)$.*

Proof. P2 will never bid above $V - R_p$ for success in any equilibrium since it is not IR for him. This implies that $P1$ will also not offer a contract which offers more than $V - R_p$ for success in period 2. Therefore payoff for G worker in period 2 is bounded above by $V - R_p$.

Suppose there was an equilibrium where G worker was getting less than $V-(R_p)/(p_g^2+(1-p_g^2)\lambda_b)$ in equilibrium. Then we can show that $P2$ can offer a little more than the maximum P1 is offering for success and get a positive payoff. By the lemma above, this improves P2's payoffs. Thus, this deviation is profitable. Therefore, original cannot be an equilibrium. □

Corollary 9. *Thus the payoffs for P1 in any equilibrium in period 2 is between $R_p(p_g^2+(1-p_g^2)\lambda_b)$ and R_p*

Proof. If G gets $V-(R_p)/(p_g^2+(1-p_g^2)\lambda_b)$ and B gets $\lambda_b(V-(R_p)/(p_g^2+(1-p_g^2)\lambda_b))$ and P2 has to get zero, then payoff for P1 = $(p_g^2+(1-p_g^2)\lambda_b)V-p_g^2(V-(R_p)/(p_g^2+(1-p_g^2)\lambda_b))-(1-p_g^2)\lambda_b(V-(R_p)/(p_g^2+(1-p_g^2)\lambda_b))=R_p$

Similarly the other one. □

Note that the above payoffs are achieved by the equilibria highlighted above.

Case 3 - Worker played N in period 1

As before in the section with one principal, the two principals will offer contracts only iff p_g is such that $p_gV+(1-p_g)\lambda_bV\geq R_p$. The worker will accept the best paying contract in period 2. The payoff in an equilibrium where both principals do offer contracts:

$$
\begin{aligned}
P1 &= 0 \\
P2 &= 0 \\
G &= V-\frac{R_p}{p_g+(1-p_g)\lambda_b} \\
B &= \lambda_b(V-\frac{R_p}{p_g+(1-p_g)\lambda_b})
\end{aligned}
$$

A.2.2 Period 1

I will show conditions needed for separation if:

1. $p_g<\frac{\lambda_b}{1-\lambda_b}\frac{R_p-\lambda_bV}{V-R_p+\lambda_bV}$.
2. $p_g\geq\frac{\lambda_b}{1-\lambda_b}\frac{R_p-\lambda_bV}{V-R_p+\lambda_bV}$.

Case 1 - $p_g<\frac{\lambda_b}{1-\lambda_b}\frac{R_p-\lambda_bV}{V-R_p+\lambda_bV}$

In this case, if the worker accepts a contract and succeeds in period, the period 2 reputation will be p_g^2 where $p_g^2<(R_p-\lambda_bV)/(V-\lambda_bV)$. This implies that the workers period 2 payoff will be 0.

Proposition 8. *Let $p_g<\frac{\lambda_b}{1-\lambda_b}\frac{R_p-\lambda_bV}{V-R_p+\lambda_bV}$. Let $V<\frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. If $p_g\in(\frac{R_p-\lambda_bR_w}{R_w-\lambda_bR_w},\frac{R_p-\lambda_bR_w+\lambda_bV(1+\delta)-R_p}{R_w-\lambda_bR_w+\lambda_bV(1+\delta)-R_p})$. Then no separating equilibrium exists.*

Proof. Note that such p_g exists if $R_p(V-R_p) < \lambda_b V(R_w - R_p)$. Also, $V < (R_w)/(1+\delta\lambda_b(2-\lambda_b))$ and $p_g < (R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)/(R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))$ imply that there exist a separating equilibrium in the principal worker model by proposition 1 in the main body of the paper.

Suppose there exists an equilibrium where G type worker forms firm in period one and B type worker accepts the best contract in period 1. Then, Competition and symmetry of principals will drive principal payoffs to zero.

Given that the equilibrium being played in period 2 if the worker accepts contract is one where the principal winning the contract offers zero wage contract which is accepted by worker and other principal offers no contract (by proposition 7), we can show that if a principal deviates and offers the following contract $\{V - R_w + \delta V, 0\}$ in period 1, then a worker of any type will accept the contract and the principal will make positive profits if $p_g > (R_p - \lambda_b R_w)/(R_w - \lambda_b R_w)$. This means that this deviation is profitable. This implies that there cannot be an equilibrium with separation. Contradiction. □

The next proposition highlights that a separating equilibrium is possible if p_g is lower.

Proposition 9. *Let* $p_g < \frac{\lambda_b}{1-\lambda_b}\frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$. *Let* $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ *and* $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}$. *If* $p_g \in (0, \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w})$ *and* $R_p > \lambda_b R_w$, *then there exists a separating equilibrium where G worker leaves to form firm in period 1 and the period 2 play is a pooling on S equilibrium.*

Proof. Let $V < (R_w)/(1+\delta\lambda_b(2-\lambda_b))$ and $p_g < (R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))/(R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p))$ guarantees that there exists a separating equilibrium in the principal-worker environment (proposition 1 in main body of this paper). A similar proof can be given to show that there exist a separating equilibrium under the above conditions in this environment. The play in period 2 after separating in period 1 is the pooling on S equilibrium. Note that we need an additional condition that $R_p > \lambda_b R_w$ to make sure that a low enough p_g exists. □

Uniqueness of Separating Equilibrium under Competitive Labour Market

We can get uniqueness of equilibrium outcome in much the same way as in the principal worker environment.

Proposition 10. *Let Assumption A and B hold. Let* $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. *There exists a* p_u *such that if* $p_g \in (0, p_u)$, *there exists a separating equilibrium in which the G worker leaves to form own firm in period one and the B worker accepts a contract in period 1. Moreover, this gives the unique equilibrium outcome of the game.*

Proof. $V < (R_w)/(1+\delta\lambda_b(2-\lambda_b))$ implies that there are no equilibria where the B type worker chooses to play L in period 1. Therefore, there are only two types of equilibrium plays are possible, either it is a separating equilibrium where the G worker leaves to form own firm or both type workers choose to accept contracts in period 1. We need to eliminate the latter type of equilibrium. Consider an equilibrium where both type workers choose to accept contracts in period 1. By assumption A and B, a G type worker can get a payoff of $V - R_w + \delta V$ by deviating and playing L. For this to be not incentive compatible, he must expect to get at least this payoff by accepting a contract in period 1. However, if $p_g < (\lambda_b)/(1-\lambda_b)(R_p - \lambda_b V)/(V - R_p + \lambda_b V)$, then the worker has to expect a payoff of zero tomorrow and neither principal can afford to offer such high wages ($V - R_w + \delta V$) in period 1. This is because they will get low prices from the customers on such contracts since the customers will pay according to their expected beliefs (which are low because p_g is low and both workers will accept any high wage offers). Therefore, we have a profitable deviation for the G type worker. Contradiction. □

Proof for Proposition 3

Following separation, let period 2 equilibrium be the pooling on S equilibrium. Then the proof follows from above.

Case 2 - $p_g \geq \frac{\lambda_b}{1-\lambda_b}\frac{R_p-\lambda_b V}{V-R_p+\lambda_b V}$

In this case, the reputation after success today is going to be such that : $p_g^2 V + (1-p_g^2)\lambda_b V - R_p > 0$.

The payoff in period 2 for the principal who signs the worker in period one will be $\lambda_b V$ if the worker fails in period 1. If the worker succeeds in period one, then by corollary 9, the payoff for the principal cannot be greater than R_p and less than $R_p(p_g^2 + (1-p_g^2)\lambda_b)$ where $p_g^2 = (p_g)/(p_g + (1-p_g)\lambda_b V)$.

Proposition 11. *Let* $p_g \geq \frac{\lambda_b}{1-\lambda_b}\frac{R_p-\lambda_b V}{V-R_p+\lambda_b V}$. *Let* $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. *If* $p_g \in (\frac{R_p-\lambda_b R_w}{R_w-\lambda_b R_w}, \frac{R_p-\lambda_b R_w+\lambda_b V(1+\delta)-R_p}{R_w-\lambda_b R_w+\lambda_b V(1+\delta)-R_p})$. *Then no separating equilibrium exists.*

Proof. Note that such p_g exists if $(1-\lambda_b)[\lambda_b V(R_w - R_p) - R_p(V - R_p)] \leq (\lambda_b V(1+\delta) - R_p)(V - R_p)$.

Suppose not. Suppose there exists an equilibrium where G type worker forms firm in period one and B type worker accepts the best contract in period 1. Competition and symmetry of principals will drive principal payoffs to zero.

Given any equilibrium that will be played in period 2, we can show that if a principal deviates and offers a contract which would pay the G worker $V - R_w + \delta V$ in total (over two periods), then that principal will make positive profits if $p_g > (R_p - \lambda_b R_w)/(R_w - \lambda_b R_w)$. This means that this deviation is profitable. This implies that there cannot be an equilibrium with separation. Contradiction. □

Proof for proposition 4

Proof follows from proposition 8,11.

Thus, there are conditions under which separating is possible in the one principal one worker model but not in the two principal one worker model.

References

Evans, D. and Leighton, L. S. (1989). Some empirical aspects of entrepreneurship. *American Economic Review*, 79(3):519–35.

Fudenberg, D. and Tirole, J. (1991). Perfect bayesian equilibrium and sequential equilibrium. *journal of Economic Theory*, 53(2):236–260.

Golan, L. (2009). Wage signaling: A dynamic model of intrafirm bargaining and asymmetric learning. *International Economic Review*, 50(3):831–854.